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ON SOME USEFUL INTERRELATIONSHIPS AMONG COMMON STABILITY PARAMETERS

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ABSTRACT

A few interrelations among common stability parameters which are useful from the computational point of view are established. Theoretical basis of the observed similarity in the behaviour of some of these parameters is explained in the light of these relationships.

Key Words : Genotype-environment interaction; concepts and measures of stability, inter-relationship, stability parameters

The success of crop improvement activities largely depends on the identification of superior varieties for mass propagation. A variety can be considered superior if it has potential for high yield under favourable environment, and at the same time has a great deal of phenotypic stability. Numerous statistics, parametric as well as non-parametric have been proposed for the measurement of yield stability [1-7]. These measures can be grouped into two categories depending upon the underlying basic stability concepts involved, *viz.* the biological and agronomic concepts of stability. The agronomic stability measures can be further classified according to whether they are based on genotype-environment (GE) interaction component or regression on environmental mean. The objective of this paper is to introduce these concepts and measures of stability and establish a few interrelationships among common stability parameters which are useful from computational point of view as well as in knowing the theoretical basis of the observed similarity in the behaviour of these parameters.

MATERIAL AND METHODS

Biological and Agronomic Concepts of Stability

Let the average phenotypic value Y_{ij} of ith genotype ($i = 1, 2, ..., t$) at the jth environment ($j = 1, 2, ..., s$) be

$$Y_{ij} = \mu + d_i + (1 + \beta_i)\, e_j + \delta_{ij} + \bar{\varepsilon}_{ij} \qquad \text{... (1.1)}$$

where μ is the general mean, d_i is the effect of ith genotype, e_j is the effect of jth environment, $(1 + \beta_i)$ is the regression of Y_{ij} on e_j, δ_{ij} is the deviation from regression for the ith genotype in the jth environment and $\bar{\varepsilon}_{ij}$ is the random error. For the genotype-environment interaction effect g_{ij} the following relationship holds:

$$g_{ij} = \beta_i e_j + \delta_{ij}$$

In model (1.1), the effects d_i, e_j, δ_{ij} are such that

$$\sum_i d_i = \sum_j e_j = \sum_i \delta_{ij} = \sum_j \delta_{ij} = \sum_{i,j} \delta_{ij} = 0$$

Further, the least square estimates of μ, d_i, e_j and $1 + \beta_j$ are:

$$\hat{\mu} = \bar{Y}_{..} = (\sum_{i,j} Y_{ij})/st$$

$$\hat{d}_i = \bar{Y}_{i.} - \bar{Y}_{..};\ \hat{e}_j = \bar{Y}_{.j} - \bar{Y}_{..}$$

$$b_i = 1 + \hat{\beta}_i = \sum_j Y_{ij}\, \hat{e}_j / \sum_j \hat{e}_j^2 \text{ and } \hat{g}_{ij} = (Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..})$$

where,

$$\bar{Y}_{i.} = (\sum_j Y_{ij}/s),\ \bar{Y}_{.j} = (\sum_i Y_{ij}/t)$$

The deviation mean squares for the ith genotype (S^2_{di}) can be defined as,

$$S^2_{d_i} = \sum_j \hat{\delta}^2_{ij}/(S - 2)$$

where $$\sum_j \hat{\delta}^2_{ij} = \sum_j Y^2_{ij} - s\bar{Y}^2_{i.} - b_i^2 \sum_j e_j^2$$

Depending upon the final goal of the breeder and the character under consideration two concepts of stability can be introduced, the biological and agronomic concepts [8]. These are also known as static and dynamic concepts respectively [9]. Under the biological concept a stable genotype is one whose phenotype Y_{ij} shows little deviation from the expected character level $\overline{Y}_{i.}$ when the genotype is performance-tested over a number of environments. This type of yield stability is analogous to the concept of genetic homeostasis, first introduced by Lerner [10]. In view of the restriction regarding constant performance level at the different environments considered, this concept is also termed the static concept of stability.

The biological concept as applied to a character like grain yield would mean a stable genotype performs well under adverse environments but not so well under favourable environments. But with increased inputs, improved technology, etc. the breeder would prefer a genotype whose performance in a particular environment is at a level expected depending on the level of productivity of the location as measured by the average productivity of all the genotypes grown in that environment. In other words he is interested in a variety which does not show any genotype-environment interaction. *i.e.* $(Y_{ij} - \overline{Y}_{i.} - \overline{Y}_{.j} + \overline{Y}_{..}) \equiv 0$, for all i. This concept which permits a predictable response in each environment and no deviation from the amount predicted is known as the agronomic or dynamic concept of stability.

An overview of the widely used parametric stability measures and their underlying stability concept is provided in Table 1. For a better appreciation of the idea conveyed through this table, we now turn to a formal definition of these measures.

Table 1. Common stability measures and their underlying stability concepts

Stability measure	Symbol	Stability concept involved
Environmental variance	$S^2_{Y_i}$	biological
Ecovalence	W_i	agronomic
Stability variance	$\hat{\sigma}^2_i$	agronomic
Regression coefficient	b_i	biological/agronomic
Deviation mean square	$S^2_{d_i}$	agronomic
Coefficient of determination	r^2_i	agronomic
Hanson's stability measure	$\hat{D}^2_{(i.)}$	agronomic

Different Measures of Stability

Most of the yield stability statistics in vogue are measures according to the agronomic concept. For the biological concept only two measures are available and they are the environmental variance $S^2_{Y_i}$ and the environmental coefficient of variation (CV_i)

$$S^2_{Y_i} = \sum_j (Y_{ij} - \overline{Y}_{i.})^2/(s-1) \qquad \text{... (2.1)}$$

$$CV_i = (S_{Y_i}/\overline{Y}_{i.}) \times 100 \qquad \text{... (2.2)}$$

lesser values of which are always sought after for higher stability. Although the measures are quite sound theoretically, they are not of much practical utility in the assessment of stability owing to the fact that (i) stability under biological concept is usually associated with relatively poor yield and (ii) high level of performance over a wide range of environments is difficult to materialize. Accordingly, these measures are rarely useful to the breeder who is always looking for high yield stability. They are however useful in disease and quality traits, the levels of which are to be maintained at all costs. We now turn to stability measures under agronomic concept.

a) Measures Based on GE Interaction Component

Wricke's ecovalence measure

The contribution of a genotype to the interaction sum of squares provides a simple and easy to compute measure known as ecovalence measure (W_i):

$$W_i = \sum_j (Y_{ij} - \overline{Y}_{i.} - \overline{Y}_{.j} + \overline{Y}_{..})^2 = \sum_j \hat{g}^2_{ij} \qquad \text{... (3.1)}$$

The lower the value of W_i the smaller will be the fluctuations from the predictable response in different environments so much so that the genotype with the least ecovalence is considered to be the ideal from the point of view of yield stability.

Shukla's [11] stability variance measure

An estimate of the variance σ^2_i of $g_{ij} + \overline{\varepsilon}_{ij}$ in terms of the residuals in a two-way classification is a useful indicator of stability of the *i*th genotype. This statistic $\hat{\sigma}^2_i$, termed stability variance is defined as follows :

$$\hat{\sigma}_i^2 = \frac{t}{(s-1)(t-2)} W_i - \frac{MS(GE)}{(t-2)} \qquad \text{... (3.2)}$$

where W_i is as defined in (3.1) and MS(GE) is the GE interaction mean square [MS(GE) = $\sum_{i,j} \hat{g}_{ij}^2/(s-1)(t-1)$]. The statistic being a linear combination of W_i, both W_i and $\hat{\sigma}_i^2$ are equivalent for the purpose of ranking the genotypes.

Hanson's stability measure

Hanson's genotypic stability measure $D_{(i.)}^2$ is defined as

$$D_{(i.)}^2 = \sum [Y_{ij} - \bar{Y}_{i.} - b_{\min}(\bar{Y}_{.j} - \bar{Y}_{..})]^2$$

$$= \sum [Y_{ij} - \bar{Y}_{i.} - b_{\min} e_j]^2 \qquad \text{... (3.3)}$$

where $b_{\min}$ is the minimum of b_i (i = 1, 2, ..., t) values in Eberhart and Russell sense. This shows that the stable genotype is one which does not deviate from the straight line

$$Y_{ij} = \bar{Y}_{i.} + b_{\min}(\bar{Y}_{.j} - \bar{Y}_{..})$$

Coefficient of determination measure

This measure of stability proposed by Pinthus [12] and symbolized as r_i^2 is defined as follows :

$$r_i^2 = \frac{b_i^2 \sum_j \hat{e}_j^2}{b_i^2 \sum_j \hat{e}_j^2 + \sum_j \hat{\delta}_{ij.}^2} \qquad \text{... (3.4)}$$

where $$\sum_j \hat{e}_j^2 = \sum (\bar{Y}_{.j} - \bar{Y}_{..})^2 \text{ and}$$

$$\hat{\delta}_{ij}^2 = \sum_j (Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..})^2 - (b_i - 1)^2 \sum_j (\bar{Y}_{.j} - \bar{Y}_{..})^2$$

$$= W_i - (b_i - 1)^2 \sum_j \hat{e}_j^2$$

the latter representing the sum of squared deviations from regression which is also symbolized as (s – 2) $S^2_{d_i}$. Unlike b_i, r_i^2 is independent of the scale of measurement. For ranking of genotypes high values of r_i^2 are regarded as being desired.

b) Measures of Stability Based on Regression on Environmental Mean

Eberhart and Russell's two-parameter measure

Eberhart and Russell [3] considered the regression coefficient, b_i as their first measure of stability:

$$b_i = \sum_j (Y_{ij} - \bar{Y}_{i.})(\bar{Y}_{.j} - \bar{Y}_{..}) / \sum_j (\bar{Y}_{.j} - \bar{Y}_{..})^2$$

$$= 1 + [\sum_j (Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..})(\bar{Y}_{.j} - \bar{Y}_{..}) / \sum_j (\bar{Y}_{.j} - \bar{Y}_{..})^2] \quad \text{... (3.5)}$$

As a second measure, they considered

$$S^2_{d_i} = [\sum_j \hat{\delta}^2_{ij}/(s-2)] - \bar{S}^2_e \quad \text{... (3.6)}$$

where $\sum_j \hat{\delta}^2_{ij}$ is as defined earlier and $\bar{S}^2_e$, the average error is given by

$\bar{S}^2_e = (\sum_j S_j^2/sr)$, S_j^2's being the error mean squares for different experiments, each conducted with the same number of replications r.

Perkins and Jinks two-parameter measure

By a slight modification of the regression technique based on Eberhart and Russell model Perkins and Jinks [4] obtained the following measures:

$$\hat{\beta}_i = \sum_j (Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..})(\bar{Y}_{.j} - \bar{Y}_{..}) / \sum_j (\bar{Y}_{.j} - \bar{Y}_{..})^2 \quad \text{... (3.7)}$$

$$S^2_{d_i} = \sum_j \hat{\delta}^2_{ij}/(s-2) \quad \text{... (3.8)}$$

where β_i is related to b_i such that $\beta_i = b_i - 1$ holds.

RESULTS AND DISCUSSIONS

Interrelationships Among Stability Measures

Interrelationships among $S^2_{Y_i}$, W_i, b_i and $S^2_{d_i}$ were pointed out by Schnell [13] as well as Wricke and Weber [14]:

$$S^2_{Y_i} = \sum_j (Y_{ij} - \bar{Y}_{i.})^2/(s-1)$$

$$= [\sum_j (Y_{ij} - Y_{i.} - \bar{Y}_{.j} + \bar{Y}_{..})^2 + \sum_j (\bar{Y}_{.j} - \bar{Y})^2$$

$$+ 2\sum (Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..})(\bar{Y}_{.j} - \bar{Y}_{..})]/(s-1)$$

$$= [W_i + \sum_j \hat{e}_j^2 + 2(b_i - 1)\sum_j \hat{e}_j^2]/(s-1) \qquad \dots (4.1)$$

in which,

$$W_i = (b_i - 1)^2 \sum_j \hat{e}_j^2 + \sum_j \hat{\delta}_{ij}^2 \qquad \dots (4.2)$$

noting that, $g_{ij} = \beta_i e_j + \delta_{ij}$, in Perkins and Jinks [4] sense, with e_j and δ_{ij} being independent of each other.

We shall now establish a few other interrelationships which are useful from computational point of view as well as in explaining the observed similarly in the behaviour of some of these parameters, when they are used for ranking purposes.

(i) Relationship of r_i^2 with other parameters:

We have

$$r_i^2 = \frac{b_i^2 \sum_j \hat{e}_j^2}{b_i^2 \sum_j \hat{e}_j^2 + \sum_j \hat{\delta}_{ij}^2} \qquad \dots (4.3)$$

$$= [(s-1)\, S^2_{Y_i} - \sum_j \hat{\delta}_{ij}^2]/(s-1)\, S^2_{Y_i}, \text{ using (4.1), (4.2)}$$

$$= 1 - \sum_j \hat{\delta}_{ij}^2 / (s-1)\, S_{Y_i}^2$$

$$= 1 - [(s-2)s_{d_i}^2 / (s-1)\, S_{Y_i}^2]$$

$$= 1 - (S_{d_i}^2 / S_{Y_i}^2), \qquad \text{... (4.4)}$$

for large s, the number of environments. Thus r_i^2 is strongly related to $S_{d_i}^2$.

Also

$$r_i^2 = \frac{1}{1 + (\sum \hat{\delta}_{ij}^2 / b_i^2 \sum_j \hat{e}_j^2)} \qquad \text{... (4.5)}$$

$$= \frac{1}{1 + [W_i - (b_i - 1)^2 \sum_j \hat{e}_j^2] / b_i^2 \sum_j e_j^2}$$

in view of Eq. (4.2). From Eq. (4.5) it is clear that r_i^2 will increase with decrease in the value of W_i especially when the variation in $(b-1)^2 \sum_j \hat{e}_j^2$ is small and accordingly rank correlation between r_i^2 and W_i is expected.

(ii) Expression of $D_{(i.)}^2$ in terms of $S_{Y_i}^2, b_i$ and $\sum_j \hat{e}_j^2$

We have

$$D_{(i.)}^2 = \sum [Y_{ij} - \overline{Y}_{i.} - b_{\min} (\overline{Y}_{ij} - \overline{Y}_{..})]^2$$

$$= \sum_j (\overline{Y}_{ij} - \overline{Y}_{i})^2 + b^{*2} \sum_j (\overline{Y}_{.j} - \overline{Y}_{..})^2 - 2b^* \sum (Y_{ij} - \overline{Y}_{i.}) (\overline{Y}_{.j} - \overline{Y}_{..})$$

$$= (s-1)\, S_{Y_i}^2 + b^* (b^* - 2b_i) \sum_j \hat{e}_j^{\,2} \qquad \text{... (4.6)}$$

where $b^* = b_{\min}$ and $\sum_j \hat{e}_j^2$ is computed using the fact that, Environment sum of squares = $t\sum_j \hat{e}_j^2$, t being the number of trial genotypes. Also making use of Eq. (4.1), we get

$$D^2_{(i.)} = W_i + 2b_i(1 - b^*)\sum_j \hat{e}_j^2 + (b^{*2} - 1)\sum_j e_j^2 \qquad \dots (4.7)$$

Thus when $S^2_{Y_i}$, b_i and the sum of squares for environment are known, this formula is convenient for the computation of $D^2_{(i.)}$ for different genotypes.

Also in view of Eqs. (4.6) and (4.7) $D^2_{(i.)}$ is expected to be highly correlated with $S^2_{Y_i}$ and b_i and moderately correlated with W_i.

(iii) Expression for $\hat{\sigma}_i^2$ in terms of $\Rightarrow S^2_{Y_i}$, b_i and $\sum_j \hat{e}_j^2$

$$\hat{\sigma}_i^2 = \frac{t}{(s-1)(t-2)} W_i - \frac{MS(GE)}{(t-2)}$$

$$= \frac{t}{(s-1)(t-2)}[(s-2)S^2_{Y_i} + \sum_j \hat{e}_j^2 - 2b_i\sum_j \hat{e}_j^2] - \frac{MS(GE)}{(t-2)} \qquad \dots (4.8)$$

using Eq. (4.1). Since $\hat{\sigma}_i^2$ is a linear function of W_i we expect perfect correlation between $\hat{\sigma}_i^2$ and W_i, also pointed out by Pham and Kang [15].

Besides the theoretical relationships, empirical correlations between stability measures are also useful in quantifying the influence of each term in the equations established above. The nature of such empirical correlations is discussed in the following section.

Empirical Correlation Between Stability Statistics

As pointed out earlier for ranking the trial genotypes, high r_i^2 values and low values of $S^2_{Y_i}$, W_i, b_i, $S^2_{d_i}$, $\hat{\sigma}_i^2$ and $D^2_{(i.)}$ are desirable. Lin *et al* [16] suggested the equivalence between statistics $\hat{\sigma}_i^2$ and W_i and between $S^2_{Y_i}$ and CV_i and proposed the use of b_i only in situations where the heterogeneity of regression accounts for a large part of total variation. Weber and Wricke [17] have reported high correlations between b_i and $S^2_{Y_i}$ and also among the three parameters $S^2_{d_i}$, W_i and r^2 and only small to moderate correlations among other combinations. High correlation between W_i and $S^2_{d_i}$ is possible only if the covariance between GE interaction and environmental effects explains only a small part of W_i as pointed out [14]. In this case the correlation between b_i and W_i will be of the same order as between b_i and $S^2_{d_i}$ and W_i with r_i^2 would indicate relatively large variability in $S^2_{d_i}$ and W_i as compared to $b_i^2 \sum_j \hat{e}_j^2$. Pham and Kang [15] expressed $\hat{\sigma}_i^2$ in the form $\hat{\sigma}_i^2 = Z_1 S^2_{d_i} + Z_2 (b_i - 1)^2 \sum_j \hat{e}_j^2 + Z_3$ where Z_1, Z_2 and Z_3 are constants and suggested this as the basis of high rank correlation between $\hat{\sigma}_i^2$ and $S^2_{d_i}$.

To study the implications of the theoretical relationships among the stability statistics and the extent of influence of each term of the equation, empirical correlations among these statistics were worked out for grain yield from multilocation wheat trials. The observed correlations are given in Table 2. Our results show highly significant correlation of r_i^2 and W_i with $S^2_{d_i}$ and $S^2_{Y_i}$ and $D^2_{(i.)}$

Table 2. Rank Correlation Coefficient among stability measures for grain yield of wheat crop

Stability measures	$S^2_{Y_i}$	W_i	$S^2_{d_i}$	r_i^2	$\hat{\sigma}_i^2$	$D^2_{(i.)}$
b_i	0.97**	0.20	–0.09	–0.40	0.20	0.99**
$S^2_{Y_i}$		0.34	0.07	–0.27	0.34	0.99**
W_i			0.68*	0.54*	1.00**	0.30
$S^2_{d_i}$				0.91**	0.68**	0.01
r_i^2					0.54*	–0.31
$\hat{\sigma}_i^2$						0.30

*, **Significant at 5%, 1% level respectively

with b_i. The correlation between r_i^2 and W_i (or $\hat{\sigma}_i^2$) is much lower than that observed between r_i^2 and $S^2_{d_i}$. This shows that the influence of the component $b_i^2 \sum_j \hat{e}_j^2$ is very little in r_i^2 whereas it is strong in the case of W_i. In fact, the role of the component

$b_i^2 \sum_j \hat{e}_j^2$ or $(b_i - 1)^2 \sum_j \hat{e}_j^2$ becomes important when the number of environments is very limited, as in the present case. This has also lowered the correlation between W_i and $S^2_{d_i}$ to some extent. Obviously the correlations of b_i with W_i and $S^2_{d_i}$ are also not of the same order. The observed high correlation of $S^2_{Y_i}$ and $D^2_{(i.)}$ with b_i can be attributed to the overwhelming contribution of 2 $b_i \sum_j \hat{e}_j^2$, a linear function of b_i to the variation in $S^2_{Y_i}$ and $D^2_{(i.)}$.

The high correlation between $D^2_{(i.)}$ and $S^2_{Y_i}$ is also expected in view of Eq. (4.6). These results lead to the conclusion that rankings based on different stability measures are comparable only if these measures are based on a large number of environments.

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