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On use of partially balanced incomplete block designs in partial diallel crosses

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Abstract

The association schemes of partially balanced incomplete block (PBIB) designs can be advantageously used for sampling the diallel crosses when the number of inbred lines involved are large. Here, the method for obtaining partial diallel cross (PDC) plans based on some existing 3-associate class association schemes alongwith some new schemes, have been discussed. The efficient plan (for a given number of lines and for a given association scheme) has been obtained based on the information per cross as compared to a complete diallel cross plan. Parameters of various PDC plans alongwith their efficiencies have been tabulated for a practical range of number of lines.

Key words: Partially balanced incomplete block design, association scheme, partial diallel cross, information per cross

Introduction

Various forms of mating designs are being used in plant and animal breeding trials to investigate the genetic properties and potentials of inbred lines or individuals. These consist of all possible single crosses (complete diallel) or a subset of all crosses (partial diallel), among a set of inbred lines. The association schemes of partially balanced incomplete block (PBIB) designs can be used to sample the crosses for a partial diallel cross (PDC) plan. A lot of work has been done in sampling the diallel crosses by utilizing two-associate class association schemes of PBIB designs [1, 2, 3 and 4]. The total number of crosses involved in a PDC using a two-associate class association scheme is likely to be large resulting in difficulty to handle all of them effectively. But for the same v (number of lines), if there is a design with higher associate class, the number of crosses are likely to be small and there is more flexibility in the choice of associate classes [3]. Some work on this aspect has been done by Narain [5], Agrawal [6], Kaushik and Puri [7] and Kaushik [8].

After getting various PDC plans, one has to choose which one is the most efficient and most suitable plan for a given situation. In this regard, not much work seems to have been done in literature. Some efficient PDC plans have been listed by Narain [5]. Das *et al.* [9] studied optimality aspects of some PDC designs.

Experiments for PDCs' are usually carried out in randomized complete block design, considering each cross as a treatment. But when the block sizes are large, considerable decrease in the efficiency of the design due to larger intra-block variances is expected. Agarwal and Das [10], Singh and Hinkelmann [4] and Sharma [11] have demonstrated the use of incomplete block designs for PDC experiments.

Here, in this study we consider partial diallel cross plans obtained through the association schemes of 3-associate class PBIB designs. The expressions for the information contained in a cross for three plans obtained from three different associates have been given. The efficiency of these plans as compared to a complete diallel cross plan has been worked out. Parameters of various PDC plans along with their efficiencies have been tabulated for number of lines ≤ 30.

Materials and methods

An incomplete block design, developed by Bose and Nair [12], is said to be partially balanced with 3-associate classes, if it satisfies the following conditions:

(i) The experimental material is divided into b blocks of k units each, different treatments being applied to units in the same block, (ii) There are v (> k) treatments each of which occurs in r blocks, (iii) There can be established a relation of association between any two treatments satisfying the following requirements:

(a) Two treatments are either 1st, 2nd or 3rd associates, the relation of association being symmetrical, (b) Each treatment has exactly n_m, mth associates (m = 1, 2, 3). (c) Given any two treatments, which are mth associates, the number of treatments common to the jth associates of the first and kth associates of the second is p^m_{jk} and is independent of the pair of treatments chosen,

(iv) Two treatments that are mth associates occur together in exactly λ_m blocks (m = 1, 2, 3).

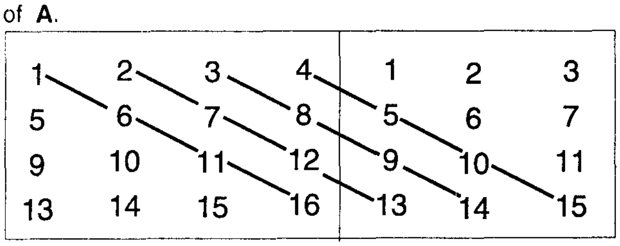
The association scheme of a 3-associate class PBIB design can be used to obtain mating designs by crossing each line with its m^{th} associates (m = 1, 2, 3). The three plans obtained are as follows:

D_1: crosses of v lines with their first associates resulting into $\frac{n_1 v}{2}$ crosses; D_2: crosses of v lines with their second associates resulting into $\frac{n_2 v}{2}$ crosses and D_3: crosses of v lines with their third associates resulting into $\frac{n_3 v}{2}$ crosses.

The generation of three plans is now illustrated through a new 3-class association scheme, named as square association scheme. We first define the scheme followed by an illustration.

Square Association Scheme: Arrange v (= s^2) distinct treatments in the form of a square array **A** of s rows and s columns. Repeat the first s-1 columns of **A** just immediately on the right hand side of it to form a rectangle of s rows and 2s-1 columns. Now, two treatments are the first associates, if they occur in the same row or same column of **A**; second associates, if they occur in the leading diagonal of **A** or in the same line parallel to the leading diagonal starting from either 2^{nd}, or 3^{rd},...., or s^{th} treatment in the first row of **A**; and third associates, otherwise. It may be noted here that in the rectangle, there are s-1 such lines parallel to leading diagonal of **A** starting from the s-1 treatments (from 2^{nd} to s^{th}) in its first row. The various parameters are $v = s^2$, $n_1 = 2(s-1)$, $n_2 = (s-1)$ and $n_3 = (s-1)(s-2)$.

Example: Let s = 4, then v = 16 treatments are arranged in the form of an array **A** of size 4 × 4. Repeating the first 3 columns of **A** on its right hand side form the rectangle of size 4 × 7. Treatments (1, 6, 11, 16) occupy positions on the leading diagonal of **A** and (2, 7, 12, 13), (3, 8, 9, 14) and (4, 5, 10, 15) respectively form different sets of treatments which occur on the three lines parallel to the leading diagonal starting from the treatments 2, 3 and 4 in the first row of **A**.



The various associates of 16 treatments are as given in Table 1.

Table 1. Different associates from a square association scheme for 16 treatments

Treatment	1st Associates	2nd Associates	3rd Associates
1	2,3,4,5,9,13	6,11,16	7,8,10,12,14,15
2	1,3,4,6,10,14	7,12,13	5,8,9,11,15,16
3	1,2,4,7,11,15	8,9,14	5,6,10,12,13,16
4	1,2,3,8,12,16	5,10,15	6,7,9,11,13,14
5	6,7,8,1,9,13	4,10,15	2,3,11,12,14,16
6	5,7,8,2,10,14	1,11,16	3,4,9,12,13,15
7	5,6,8,3,11,15	2,12,13	1,4,9,10,14,16
8	4,12,16,5,6,7	3,9,14	1,2,10,11,13,15
9	10,11,12,1,5,13	3,8,14	2,4,6,7,15,16
10	9,11,12,2,6,14	4,5,15	1,3,7,8,13,16
11	9,10,12,3,7,15	1,6,16	2,4,5,8,13,14
12	9,10,11,4,8,16	2,7,13	1,3,5,6,14,15
13	14,15,16,1,5,9	2,7,12	3,4,6,8,10,11
14	13,15,16,2,6,10	3,8,9	1,4,5,7,11,12
15	13,14,16,3,7,11	4,5,10	1,2,6,8,9,12
16	13,14,15,4,8,12	1,6,11	2,3,5,7,9,10

Considering the treatments as lines, the three different PDC plans (D_1, D_2 and D_3) obtained by crossing each line with its m^{th} associates (m = 1, 2, 3) are given in Table 2.

Table 2. PDC plans obtained using square association scheme for 16 lines

Plan D_1	Plan D_2	Plan D_3
1×2, 1×3, 1×4, 1×5,	1×6, 1×11,	1×7, 1×8, 1×10, 1×12,
1×9, 1×13, 2×3, 2×4,	1×16, 2×7,	1×14, 1×15, 2×5, 2×8,
2×6, 2×10, 2×14, 3×4,	2×12, 2×13,	2×9, 2×11, 2×15, 2×16,
3×7, 3×11, 3×15, 4×8,	3×8, 3×9,	3×5, 3×6, 3×10, 3×12,
4×12, 4×16, 5×6, 5×7,	3×14, 4×5,	3×13, 3×16, 4×6, 4×7,
5×8, 5×9, 5×13, 6×7,	4×10, 4×15,	4×9, 4×11, 4×13, 4×14,
6×8, 6×10, 6×14, 7×8,	5×10, 5×15,	5×11, 5×12, 5×14,
7×11, 7×15, 8×12,	6×11, 6×16,	5×16, 6×9, 6×12, 6×13,
8×16, 9×10, 9×11,	7×12, 7×13,	6×15, 7×9, 7×10, 7×14,
9×12, 9×13, 10×11,	8×9, 8×14,	7×16, 8×10, 8×11,
10×12, 10×14, 11×15,	9×14,	8×13, 8×15, 9×15,
12×16, 13×14, 13×15,	10×15,	9×16, 10×13, 10×16,
13×16, 14×15, 14×16,	11×16,	11×13, 11×14, 12×14,
15×16	12×13	12×15

In this example, out of the three PDC plans, through Plan D_1 and D_3 all possible paired general combining ability (*gca*) effects can be estimated. It can also be seen that the required number of crosses has been reduced to 48 in both the cases from a required total of 120 in case of a complete diallel.

Model: The following fixed effects model involving the *gca* effects of lines is being considered:

$$Y_{ii'} = \mu + g_i + g_{i'} + e_{ii'} \qquad \text{... (1)}$$

$$i < i' = 1, 2, ..., v$$

where $Y_{ii'}$ is the response obtained from the cross (i x i′), μ is the general mean and g_i ($g_{i'}$) is the *gca* effect of line i(i′), $e_{ii'}$'s are considered to be independent random variables with mean zero and variance σ^2. In matrix notation

$$\mathbf{Y} = \mu\mathbf{1} + \mathbf{Xg} + \mathbf{e} \qquad \text{... (2)}$$

where **Y** is the column vector of $\frac{n_m v}{2}$ observations (m = 1, 2, 3) depending on the PDC plan D_1, D_2 or D_3 considered, **1** is a $\frac{n_m v}{2} \times 1$ vector of unity, $\mathbf{g}' = (g_1, g_2, ..., g_v)$, $\mathbf{X} = ((x_{\alpha i}))$, the $\frac{n_m v}{2} \times v$ design matrix with $x_{\alpha i} = 1$, if i^{th} line occurs in the α^{th} cross ($\alpha = 1, ..., \frac{n_m v}{2}$) and $x_{\alpha i} = 0$ otherwise and **e** is a $\frac{n_m v}{2} \times 1$ vector of errors.

There will be three variances for testing the estimated differences $(\hat{g}_i - \hat{g}_{i'})$ according as the parents i and i′ are first, second or third associates, from each D_m, m = 1, 2, 3. Following Narain [5], the average variance of the difference between *gca* effects for the m^{th} plan can be obtained as follows:

$$\bar{V}D_m = \frac{\frac{vn_1}{2} V_{m1} + \frac{vn_2}{2} V_{m2} + \frac{vn_3}{2} V_{m3}}{\frac{v(v-1)}{2}}$$

$$= \frac{n_1 V_{m1} + n_2 V_{m2} + n_3 V_{m3}}{n_1 + n_2 + n_3} \quad \text{... (3)}$$

where V_{mm} is the variance of the difference between *gca* effects of two lines that are crossed, $V_{mm'}$ ($m \neq m' = 1, 2, 3$) is the variance between *gca* of two lines that are not crossed. These variances are obtained from the inverse of the information matrix for *gca* effects. The information obtained per cross from the m^{th} plan is

$$\text{Inf}_m = \frac{2}{vn_m \bar{V}D_m} \quad \text{... (4)}$$

The efficiency factor in terms of the information per cross as compared to a complete diallel cross plan, assuming the error variance to be same for both the plans, is worked out as follows:

$$E_m = \frac{n_m \bar{V}D_m}{\sum_{m=1}^{3} n_m \bar{V}} \quad \text{... (5)}$$

where $\bar{V}$ is the average variance of the estimate of the elementary contrast pertaining to *gca* effects in case of a complete diallel cross plan.

Table of PDC plans

A table of PDC plans has been prepared and is given in Annexure for number of lines ≤ 30. The plans are obtained by using eleven different three-associate class association schemes namely, Rectangular, RTR (v = pxq) [13], Extended triangular, ETR (v = (p+2)(p+3)(p+4)/6) [14], Cubic, CBC ($v = p^3$) [15], Circular, CIR (v = pxq) [16, 17], Circular Lattice, CRL ($v = 2xp^2$) [18], Nested Group Divisible, NGD (v = pxqxl) [19], Nested triangular, NTR (v = pxqx(q-1)/2) [20], Nested L_2, NL2 ($v = pxq^2$) [20], Triangular (3), TR3 (v = sxv*) [20], Square, SQR ($v = s^2$) [20] and Sector, STR ($v = 2xmxn^2$) [20]. The details of all these 11 association schemes can be found in [20].

The table contains number of lines (v) alongwith its structure, number of m^{th} associates (n_m, m = 1,2,3), number of crosses (s_m), information per cross for PDC plans (Inf_m), information per cross for complete diallel cross plan (Inf_{CDC}) and association scheme used for obtaining PDC plans. The efficiency of these plans have been worked out as compared to a complete diallel plan and the efficiency (E) corresponding to the most efficient plan, among the three plans, permitting the estimability of all possible paired *gca* effects has been reported.

It is seen from the table that the efficiencies of all the plans is more than 65%. As the number of crosses increase, these efficiencies show an increasing trend. Large number of plans has efficiency more than 90% and so one can easily experiment with the sample of crosses instead of all possible crosses. This will save the resources without causing much decrease in accuracy of the estimates pertaining to *gca* effects of the lines involved. In some cases, for a given number of lines the breeder has different choices for selecting the crosses based on various association schemes. A SAS code has been prepared using PROC IML module [20] that generates the information matrix, the three types of variances and the efficiencies. The plan with the maximum efficiency is selected as the efficient mating design, for a given number of lines. The most efficient PDC plan can then be laid out in a suitable environmental design. The method of analysis considering PBIB(3) design as an environmental design can be easily worked out from Singh and Hinkelmann [4].

A computer software for PBIB(3) designs and partial diallel crosses has been developed which is capable of generating efficient cost effective plans for partial diallel crosses obtained through three-class association schemes and carrying out its analysis in complete or incomplete block settings [21].

Conclusion

The association schemes of PBIB(3) designs as used for generating plans for PDC have been discussed and efficiency of these plans have been evaluated. A list of efficient plans have been prepared which will be quite useful for plant breeders in sampling the diallel crosses.

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ANNEXURE: Table of Parameters of Efficient PDC Plans

v	n_1	n_2	n_3	s_1	S_2	S_3	Inf_1	Inf_2	Inf_3	Inf_{CDC}	E	Scheme
$8(=2^3)$	3	3	1	12	12	4	0.1296	0.0931	0.0583	0.1071	0.8693**	CBC
9(=3×3)	2	2	4	9	9	18	0.0684	0.0684	0.0794	0.0972	0.8169***	RTR
9(=3×3)	4	2	2	18	9	9	0.0794	0.0684	0.0684	0.0972	0.8169*	SQR
12(=4×3)	2	3	6	12	18	36	0.0524	0.0632	0.0672	0.0758	0.8865***	RTR
15(=5×3)	2	6	6	15	45	45	0.0424	0.0424	0.0424	0.0561	0.7558**	CIR
15(=5×3)	2	4	8	15	30	60	0.0424	0.0549	0.0568	0.0619	0.9176***	RTR
16(=4×4)	3	3	9	24	24	72	0.0481	0.0481	0.0524	0.0569	0.9209***	RTR
16(=4×4)	6	3	6	48	24	48	0.0521	0.0481	0.0521	0.0583	0.8937***	SQR
18(=6×3)	2	5	10	18	45	90	0.0356	0.0478	0.0488	0.0523	0.9331***	RTR
18(=3×2×3)	2	3	12	18	27	108	0.0356	0.0726	0.0497	0.0523	0.9503***	NGD
$18(=2\times3^2)$	1	8	8	9	72	72	0.2361	0.047	0.0466	0.0523	0.8987**	CRL
$18(=2\times3^2)$	1	8	8	9	72	72	0.2361	0.0468	0.0468	0.0523	0.8942**	STR
20(=5×4)	3	8	8	30	80	80	0.0388	0.0352	0.0352	0.0474	0.7426**	CIR
20(=5×4)	3	4	12	30	40	120	0.0388	0.0416	0.0454	0.0474	0.9578***	RTR
20(=(4×5×6)/6)	9	9	1	90	90	10	0.0446	0.0425	0.2000	0.0466	0.9571*	ETR
20(=2×(5×4)/2)	1	12	6	10	120	60	0.2111	0.4403	0.4043	0.0473	0.9313**	TR3
21(=7×3)	2	6	12	21	63	126	0.0307	0.0421	0.0428	0.0452	0.9469***	RTR
24(=3×8)	7	2	14	84	24	168	0.0376	0.0270	0.0380	0.0398	0.9548***	RTR
24(=6×4)	3	5	15	36	60	180	0.0325	0.0362	0.0386	0.0399	0.9674***	RTR
24(=3×2×4)	3	4	16	36	48	192	0.0325	0.0504	0.0383	0.0399	0.9599***	NGD
24(=4×2×3)	2	3	18	24	36	216	0.0270	0.0548	0.0391	0.0399	0.9800***	NGD
25(=5×5)	4	10	10	50	125	125	0.0335	0.0300	0.0300	0.0383	0.7833**	CIR
25(=5×5)	4	4	16	50	50	200	0.0335	0.0335	0.0373	0.0383	0.9739***	RTR
25(=5×5)	8	4	12	100	50	150	0.0354	0.0335	0.0365	0.0383	0.9530***	SQR
27(=9×3)	2	8	16	27	108	216	0.0241	0.0339	0.0344	0.0356	0.9663***	RTR
27(=3×3×3)	2	6	18	27	81	243	0.0241	0.0311	0.0344	0.0356	0.9663***	NGD
$27(=3\times3^2)$	4	4	18	54	54	243	0.0278	0.0278	0.0344	0.0356	0.9663***	NL2
$27(=3^3)$	6	12	8	81	162	108	0.0319	0.0331	0.0314	0.0357	0.9272**	CBC
$27(=3\times3^2)$	2	12	12	27	162	162	0.0241	0.0330	0.0333	0.0356	0.9339***	STR
28(=7×4)	3	6	18	42	84	252	0.0280	0.0318	0.0335	0.0344	0.9738***	RTR
30(=5×6)	5	12	12	75	180	180	0.0291	0.0262	0.0261	0.0321	0.8162**	CIR
30(=6×5)	4	5	20	60	75	300	0.0280	0.0291	0.0316	0.0322	0.9814***	RTR
30(=3×(5×4)/2)	6	3	20	90	45	300	0.0286	0.0190	0.0312	0.0322	0.9689***	NTR
30(=3×2×5)	4	5	20	60	75	300	0.0280	0.0387	0.0302	0.0322	0.9379***	NGD

*Obtained using first associates; **Obtained using second associates; ***Obtained using third associates